FID 200 Modern Physics

Problem Set 2

Schrödinger Equation and Infinite Square Well Potential

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1. Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, σ_x and σ_p for the nth stationary state of the infinite square well ($n = 1$ is the ground state). Check that the uncertainty principle is satisfied. Which state comes close to the uncertainty limit?

2. A particle in the infinite square well has as its initial wave function an even mixture of the first two stationary states:

$$\Psi(0, x) = A\ (\psi_1(x) + \psi_2(x))$$

   (a) Normalize $\Psi(0, x)$ – that is, find A!

   (b) Find $\Psi(t, x)$ and $|\Psi(t, x)|^2$. Let $\omega \equiv \pi^2 \hbar / (2\, m\, a^2)$ (Express $|\Psi(t, x)|^2$ in terms of sinusoidal functions of time, eliminating the exponentials with the help of $\exp(\Im\, \theta) = \cos(\theta) + \Im\ \sin(\theta)$.)

   (c) Compute $\langle x \rangle$ and $\langle p \rangle$.

   (d) Compute $\langle \mathcal{H} \rangle$. How does it compare with E_1 and E_2.

3. A particle in the infinite square well has the initial wave function

$$\Psi(0, x) = \begin{cases} A\, x & \text{for} \quad 0 \le x \le a/2\ , \\ A\, x\, (a - x) & \text{for} \quad a/2 \le x \le a \end{cases}$$

   (a) Sketch $\Psi(0, x)$ and determine the constant A.

   (b) Find $\Psi(t, x)$. **Clue:** Use Equation (3.20) of my lecture note. You may determine c_n with "Fourier trick" that is explained in Equation (3.38) of my lecture note. Please aware of the integral boundaries.

   (c) What is the total energy expression E_n where index n is the state of particle.

(d) What is the probability that a measurement of the energy would yield the value of E_1?

(e) Find the expectation value of the energy. **Clue:** You may simplify the series $1 + 1/3^2 + 1/5^2 + 1/7^2 + \ldots \approx \pi^2/8$.

4. A particle of mass m in the infinite square well (of width a) starts out in the left half of the well, and is (at $t = 0$) equally likely to be found at any point in that region.

(a) What is its initial wave function, $\Psi(0, x)$? **Clue:** Assume it is real, do not forget to normalize it.

(b) What is the probability that a measurement of the energy would yield the value $\pi^2 \hbar^2/(2\, m\, a^2)$?

5. A particle in the infinite square well has the initial wave function

$$\Psi(0, x) = A \, \sin^3\left(\frac{\pi}{a}\, x\right) \qquad \text{for} \quad 0 \le x \le a \, .$$

Determine A, find $\Psi(t, x)$, and calculate $\langle x \rangle$, as a function of time. What is the expectation of the energy? **Clue:** $\sin(3\,\theta) = 3 \, \sin(\theta) - 4 \, \sin^3(\theta)$.